A Technology-based Mathematics Enrichment Program Synergising Singapore's Concrete-Pictorial-Abstract Approach and Math Circles

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Introduction

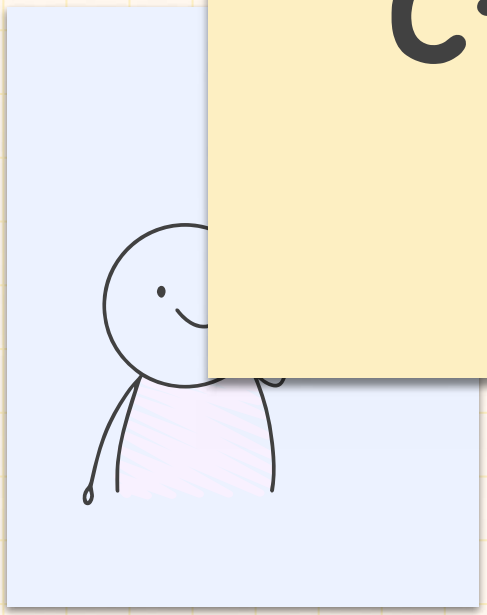
Classroom Context (based in the San Francisco Bay Area):

- TK - 6th Grade, 680 students
- 2 administrators, 28 teachers
- 45% English-Language Learners
- 95% Minority Enrollment
    - High population of Filipino and Hispanic families
- 30% Low-Income Households
- On the 2021 - 2022 CAASPP (California Assessment of Student Performance and Progress)
    - Spangler's Proficiency in ELA = 72%
      CA's Proficiency in ELA = 47%
    - Spangler's Proficiency in Math = 61%
      CA's Proficiency in Math = 33%
- Integrated the C-P-A approach into my classroom to encourage students to use concrete experiences to construct knowledge, communicate their reasoning, and relate it to real world mathematics

Students whose mathematical learning includes **manipulative experiences** will be more likely to bridge the gap between the world in which they live and the abstract world of mathematics.

— Dr. Zoltan Dienes, 1960

C-V and V-P-A Lessons in Singapore



Bar Graph Lesson
Low-Progress Primary 3 Class

- Students used concrete manipulatives to 1) explain the relationship between picture graphs and bar graphs and 2) compare two values by physically taking post-its on and off.
- Level of questioning in the Abstract
  - Reading **the** graph - “How many students like Swimming?”
  - Reading **between** the graph - “How many more students like Swimming compared to Badminton?”
  - Reading **beyond** the graph - “If eight total students like Soccer and three of those students are girls, how many boys like Soccer?”
- **12 out of 16 students** were able to achieve **all three** lesson objectives in the final assessment worksheet:
  - read and interpret data from bar graphs **using mathematical language**
  - compare data from bar graphs
  - make overall conclusions about the purpose of bar graphs
- **4 out of 16 students** struggled with comparing data. However, all 4 students were successful after **more concrete experience** with post-its and virtual unifix cubes.

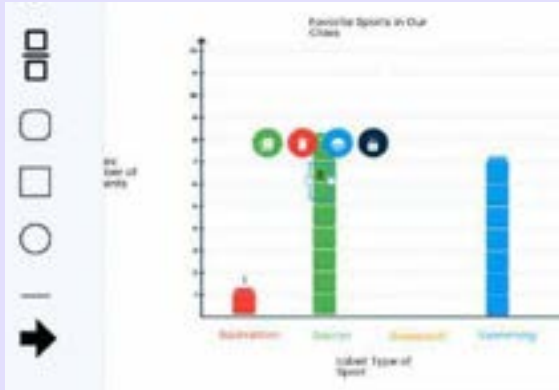
Why Use the C-V and V-P-A Approach?

Effectiveness:

- The unifix cubes automatically snapped together and snapped apart (instead of requiring students to draw or erase a separate box for each segment of a bar).
- The unifix cubes ensured that each column in the bar graph had equal width (instead of requiring students to measure with a ruler to draw columns with equal width).
- “I prefer the virtual bar graph because I like to label the number on top of the bar to know the exact value for each bar.” This helped students recognize patterns or trends.

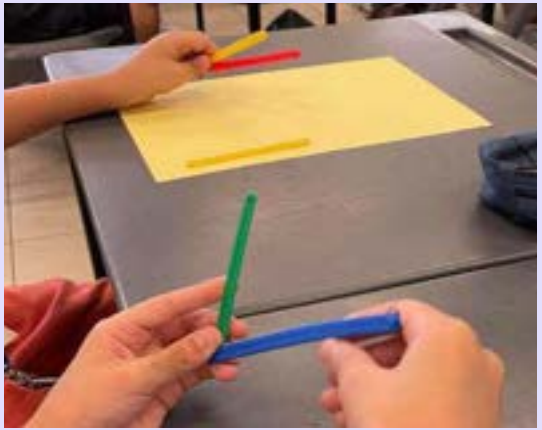
Efficiency:

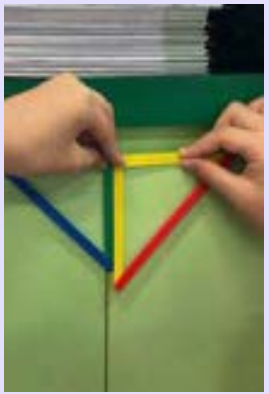
- The virtual manipulatives were accessed quickly on the iPads by typing in one code. No time was spent distributing and retrieving concrete unifix cubes or post-its.
- Students were able to construct the virtual model quickly and efficiently compared to the concrete model of individually constructing, connecting, and drawing around the post-its.



Angles Lesson
Mixed-Ability Primary 3 Class

- Students used mixed media to 1) discover the **Big Idea of Measures** (associating angles with the amount of turning) and 2) explore right angles in their environment.
- Students showed high engagement and interest when discussing their reasoning with peers because they were able to link new, abstract information (a right angle) to already solidified networks of knowledge.
- Students were **metacognitive** in analyzing the task and reflecting on their learnings and previous misconceptions.
    - **Originally 17 out of 33 students** believed the size of an angle was dependent on the lengths of its arms
    - **After manipulative experiences, all 33 students** were able to reconstruct correct conceptions between arm length and degree of angle
- **29 out of 33 students** were able to achieve **all three** lesson objectives in the final virtual geoboard assessment:
    - Identify angles in the classroom and environment
    - Associate an angle with a certain amount of turning
    - Form a right angle





Why use the C-V and V-P-A approach?

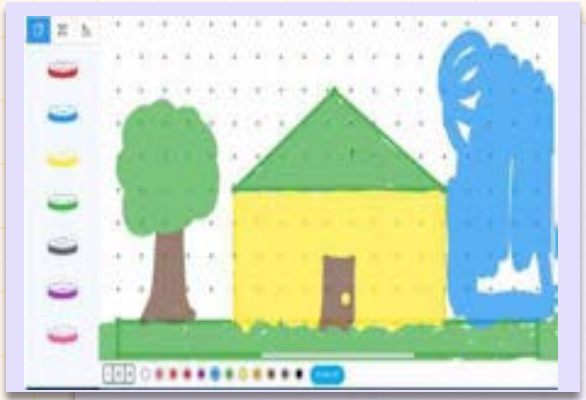
Effectiveness:

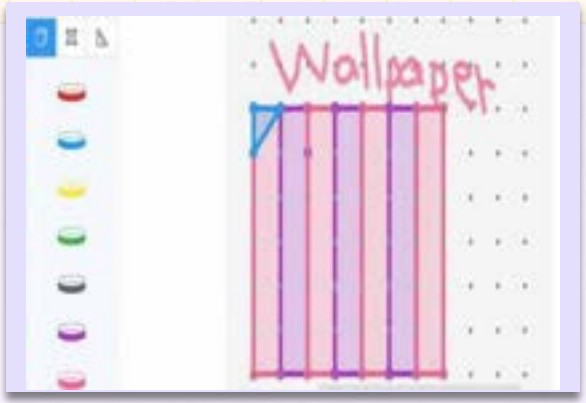
- Students were **only able to construct polygons** on 11x11 virtual geoboards.
- Virtual geoboards encouraged students to be creative and have fun in meaning-making.
    - There was **student choice** and **student-driven discovery** based on individualized needs (e.g. drawing simple polygons vs. more complicated designs).

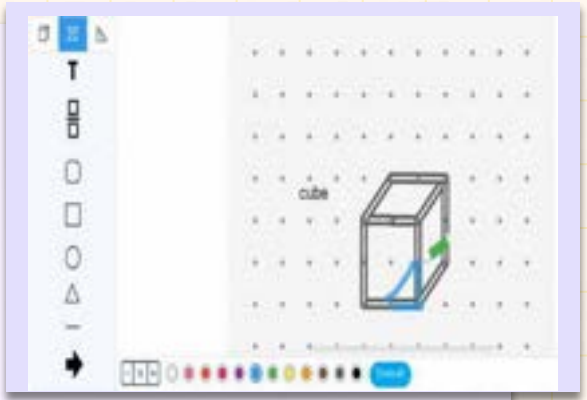
Efficiency:

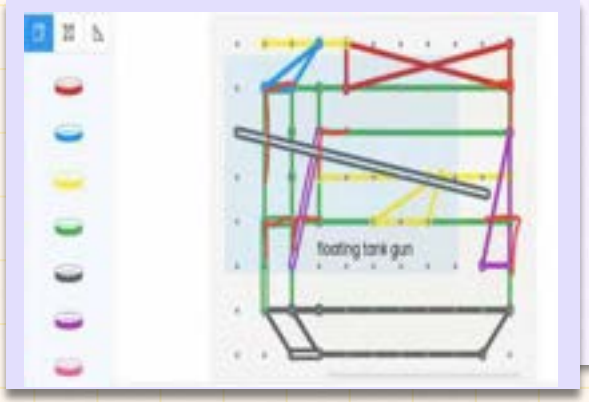
- The geoboard was accessed quickly on the iPads by typing in one code. No time was spent distributing and retrieving physical geoboards and rubber bands. In addition, the construction of the polygons was less time-consuming since there was no fine motor skill requirement to apply/take-off rubber bands.
- Students used supplies appropriately and focused on the task at hand (instead of playing around with physical rubber bands).
- The intuitive interface of the virtual geoboard required low cognitive load so students could focus on the problem task.
- Students highlighted right angles directly on virtual geoboards and used mathematical language to justify their choices and get immediate feedback from peers and the teacher.

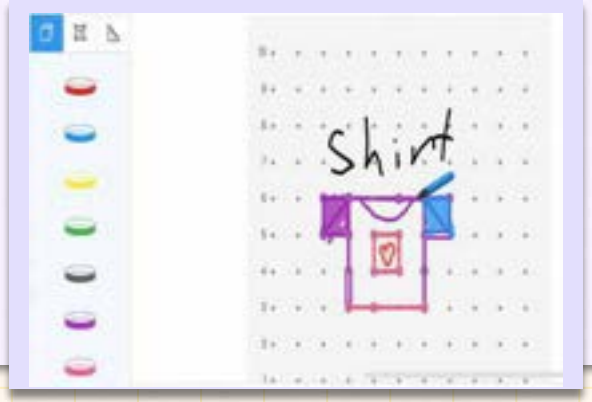
Virtual Geoboard Designs

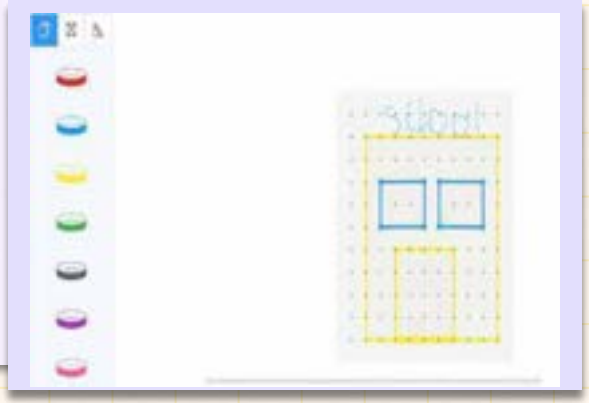






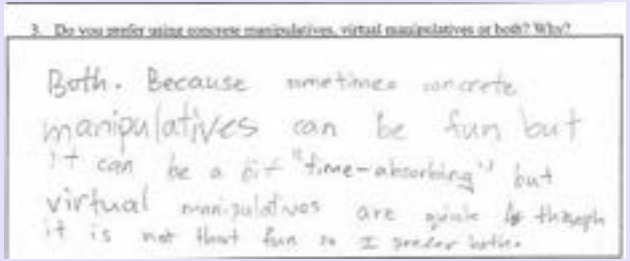


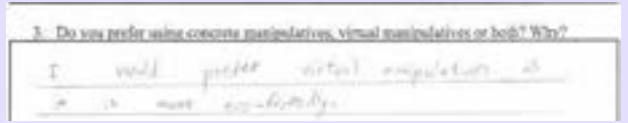




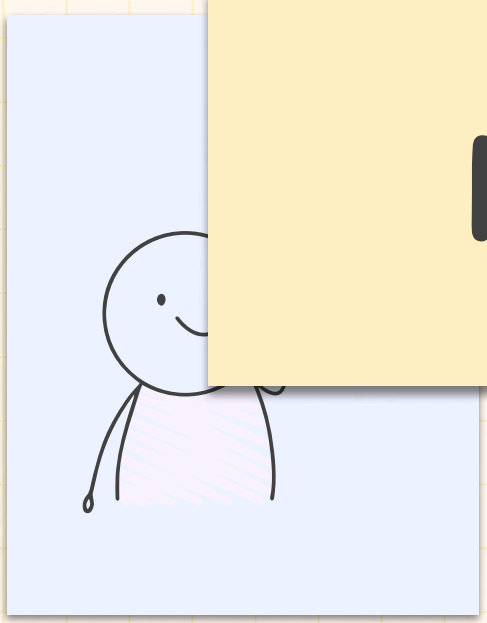
Limitations and Considerations

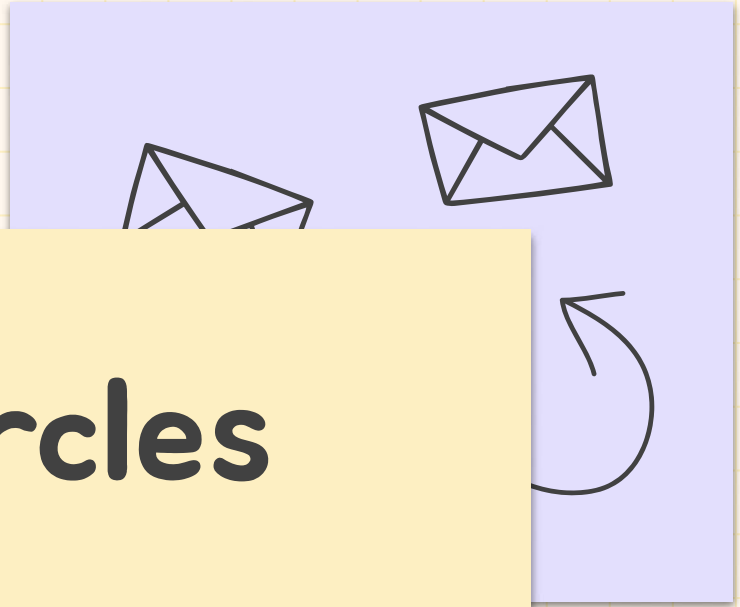
- Key Applications of Technology (KAT)
- Using virtual manipulatives as a supplement, complement, or in lieu of concrete manipulatives
    - “Once conceptual understanding is effected with concrete manipulative, the subsequent use of virtual manipulatives seems to facilitate bridging to the abstract” (Hunt et al., 2011, p.6)
    - Providing opportunities for 3D representational awareness since nearly everything students interact with in their daily lives is in 3D
- Choosing the most appropriate virtual manipulatives
- Understanding the particular group of students and their abilities in thinking, problem solving, metacognitive skills
- Factoring in students’ development of fine motor skills, muscle control, spatial thinking and reasoning, hand-eye coordination, directional awareness
- Understanding students’ familiarity with technology
    - Synchronous vs. Asynchronous Instruction





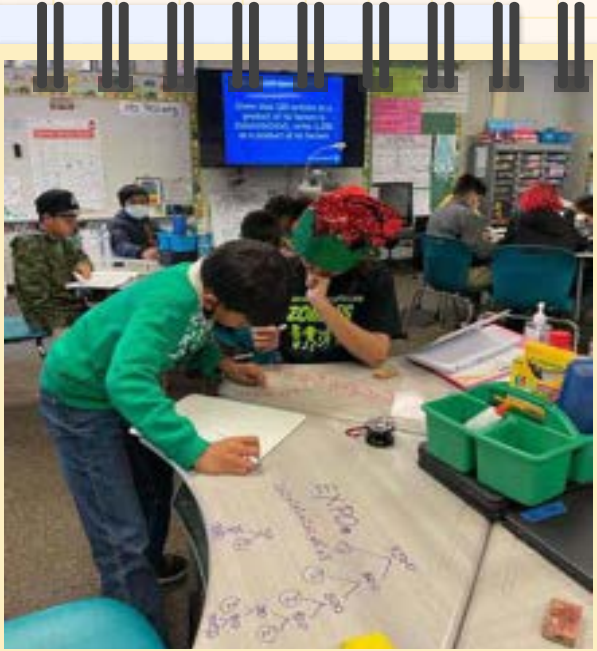
Math Circles





What are Math Circles?

- A **social structure** where students engage collaboratively in the depths and intricacies of mathematical thinking and cultivate a culture of doing mathematics
- Focused on the enjoyment of mathematical problem solving and the building of mathematical habits of mind (Burns et al., 2017)
- Covered topics not normally included in standard classroom curriculum
- Engaged teachers, professionals from all sectors, and community members
- Conducted in informal settings, such as after-school programs and professional learning communities
- Included methods such as storytelling, experiments, hands-on activities, independent work, teamwork, guided discussions, handouts, and competitions



Students construct viable arguments and critique others' reasoning during Jeopardy competitions (MHM3).

My Math Circle

- Held for an 8 week period, weekly meetings after-school
- Included 15 mixed-ability fifth grade students **(with a focus on underrepresented and underserved students)**
- Highlighted the importance of mathematics being a **growth subject** (Boaler, 2016)
- Cultivated mathematically curious and math-loving kids with improved attitudes and motivation
- Used inquiry-based tasks to promote mathematical reasoning and problem solving (less rote math, more hands-on activities with multiple representations)

Math Circle Activity: Girl Scout Cookies

Your task is to figure out how many boxes of Girl Scout cookies will fit into the trunk of a Nissan Rouge.

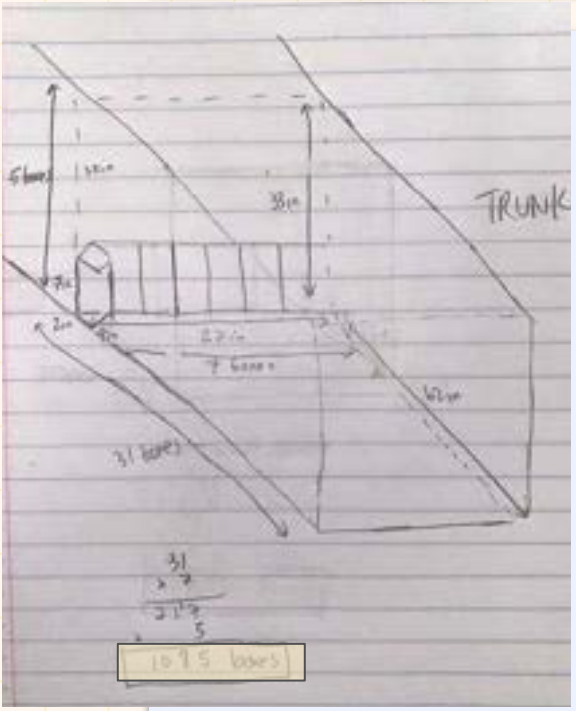
62.3 in x 27.1 in 33.25 in = 56,136.97 cubic inches
56,136.97 cubic inches/1728 = **32.48 cubic feet**

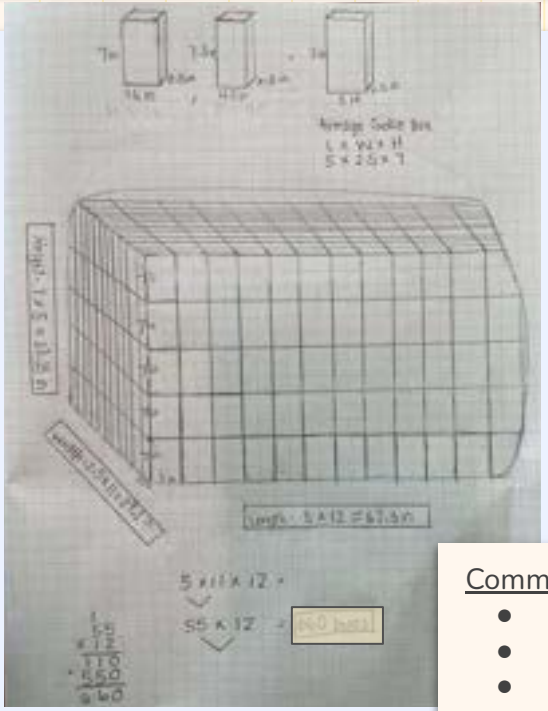
Task involves:

- Estimation
- Spatial reasoning, logical deduction and reasoning
- C-P-A approach or C-V and V-P-A approach

*adapted from Dan Meyer

Students' Pictorial and Abstract Work





(5·6·1)·18 layers = 540 boxes for half of the trunk

540 · 2 halves = 1080 boxes total in the entire trunk

Common Core Mathematical Practices targeted:

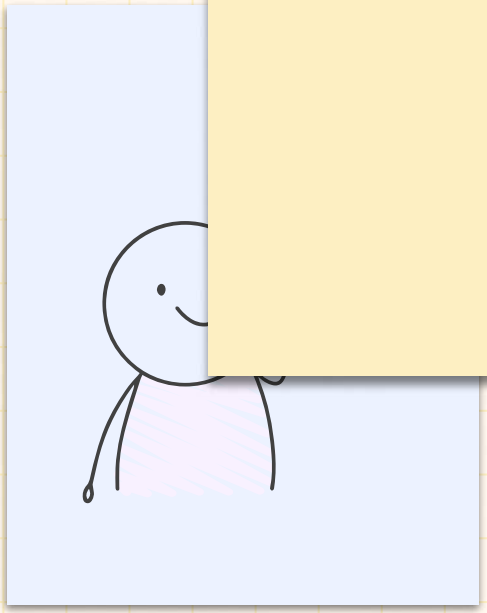
- **MP1** - Make sense of problems & persevere in solving them.
- **MP2** - Reason abstractly and quantitatively.
- **MP4** - Model with mathematics.
- **MP5** - Use appropriate tools strategically.
- **MP6** - Attend to precision.
- **MP7** - Look for and make use of structure.

Student Outcomes

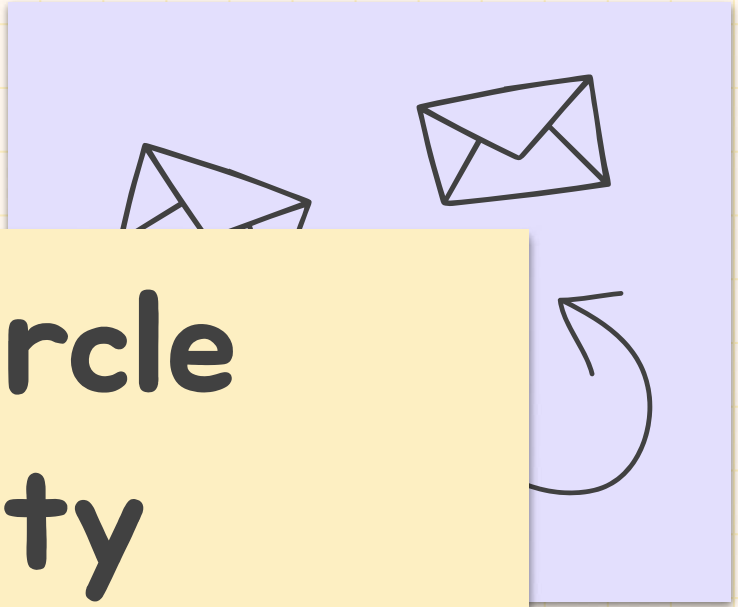
Students were able to use the C-P-A approach to **estimate** the total volume of a 3D object/space, along with how many Girl Scout cookie boxes would fit in that space. This authentic task encouraged students to 1) take accurate measurements and 2) use those measurements to make prudent decisions on material quantity, material costs, project feasibility, and the overall efficiency of the task.

Student Observations and Conclusions:

- The Girl Scout cookie boxes had different measurements depending on the type of cookie. Each group decided what final measurements to use.
- Students used different orientations (e.g. standing upright, lying flat) of the boxes.
    - Demonstrating 3D shapes in different orientations and their spatial relations via static and dynamic representations through technology may assist students in constructing geometric knowledge (Battista, 2007; Guven, 2012).
- **Our staff secretary's 2018 Nissan Rouge S had a different trunk dimension (32 cubic feet) than the 2015 model listed in the video (39.3 cubic feet).**
- The boxes could not fit into every nook and cranny of the car.



Math Circle Activity



Math Circle Activity: Skyscrapers

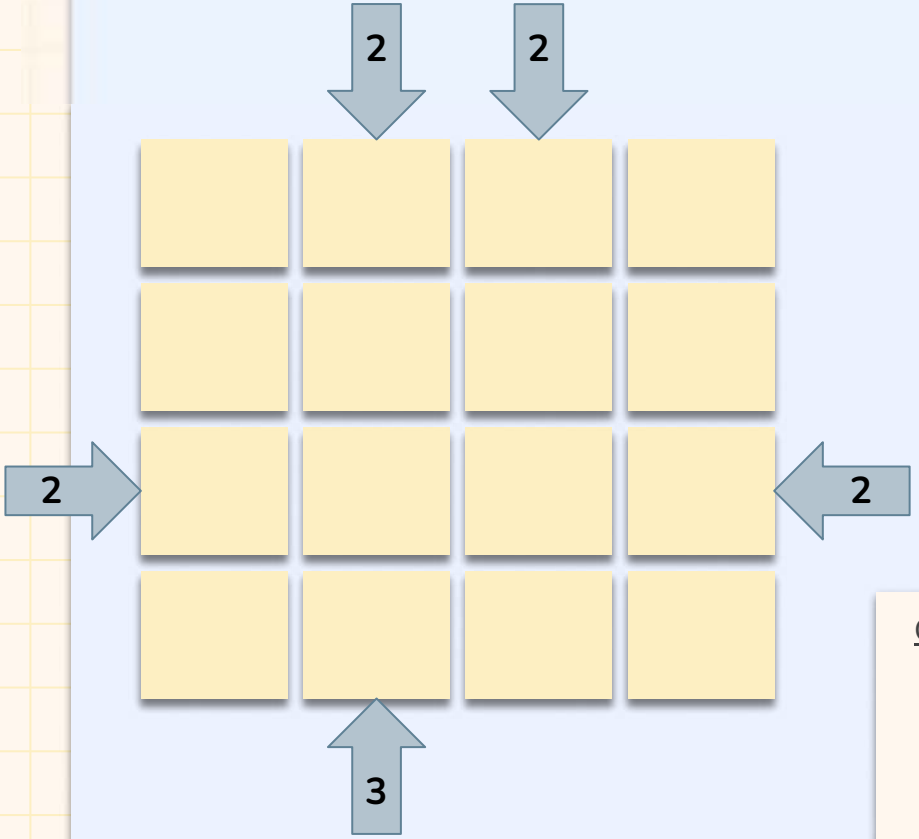
Your task is to help a city planner figure out where to build skyscrapers according to certain rules:

1. Each square must have a skyscraper.
2. Each row and column must have skyscrapers of **different** heights (1 through 4).
3. The number outside the grid tells you how many skyscrapers you can see from that direction.
4. Taller skyscrapers block out the view of shorter skyscrapers located behind them.

Task involves:

- Spatial reasoning, logical deduction and reasoning
- Pattern recognition (combinatorics)
- C-P-A approach or C-V and V-P-A approach
- Scaffolding (in materials and activity tasks)

*adapted from MathCircles.org & BrainBashers



Common Core Mathematical Practices targeted:

- **MP1** - Make sense of problems & persevere in solving them.
- **MP5** - Use appropriate tools strategically.
- **MP7** - Look for and make use of structure.
- **MP8** - Look for and express regularity in repeated reasoning.

Concrete Experience Worksheet
Virtual Manipulative (https://jrmf.org/puzzle/skyscrapers/)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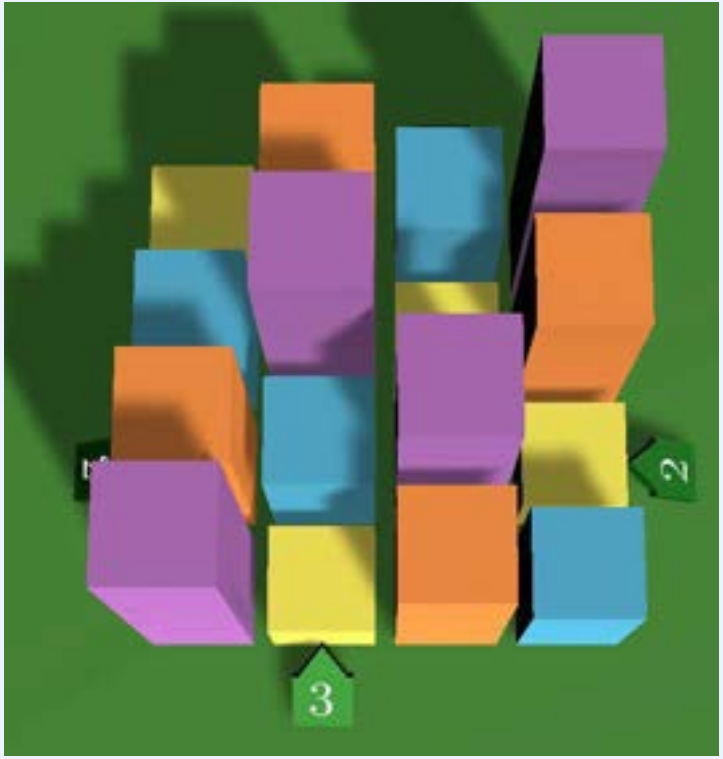
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Questions to Consider

Teacher Questions:

1. When will you discuss the learning goal (at the onset, during the closing, etc)?
2. Do you start with context, visuals, questions, and/or explanations?
3. Do you provide strategies and “show” first or allow time for productive struggle?
4. Are all students **required** to use manipulatives or will it be offered only for those who want it?
5. Do students have free choice over when technology becomes a direct tool substitute?
6. How would you monitor student thinking (through observations, conversations, and/or product)? What assessing questions might you ask?

Student Questions:

1. How do you start a puzzle?
2. Have you found a strategy that works for many or all of the puzzles? Tell me more.
3. Did you keep track of spots you could or could not place a skyscraper?
4. **Extension:** Ask students to craft their own Skyscraper puzzles for the class to solve.

Math Circle Resources

Math Circle Activities and Inquiry-based Tasks:

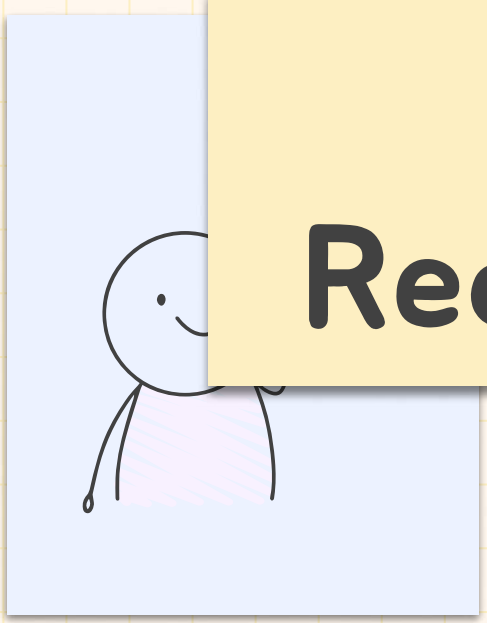
1. Inside Mathematics (University of Texas at Austin)
2. MARS (Mathematics Assessment Resource Service)
    a. Bowland Maths Assessment Tasks
3. MathCircles.org
    a. SF Math Circle Activities
4. NRICH (University of Cambridge)
    a. Primary Curriculum Map
5. Youcubed (Stanford University)

Educator Blogs:

1. Three-Act Math Tasks (Dan Meyers)
2. Numberless Word Problems (Brian Bushart)
3. Problem Based Lessons (Robert Kaplinsky)

Thank you.

Please reach out to ilinghsiung@gmail.com if you have further questions.



Technology Recommendations

Interactive Platforms

Google Workspace

Nearpod

Google Workspace

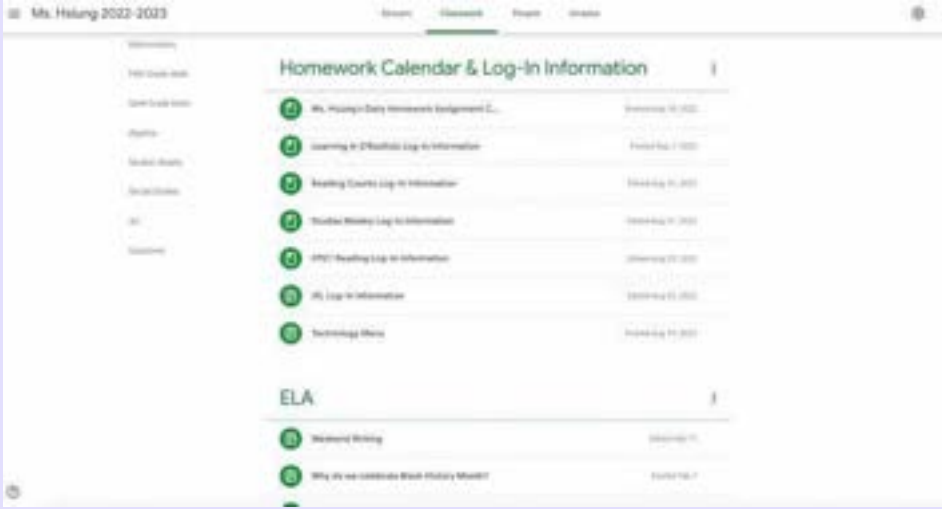
Google Classroom helps us do assignments and turn in work on time. I can talk to my friends on **Google Chat** and help when they have questions about something Ms. Hsiung taught in class. Last, I like to type essays on **Google Docs or Google Slides** instead of writing on paper.
- Daniel Bacungun (5th Grade)

I use **Google Meets** to get together with my friends to work on group projects. I have also used it to meet with teachers and family members during Covid-19.
- Aaron Ramos (6th Grade)

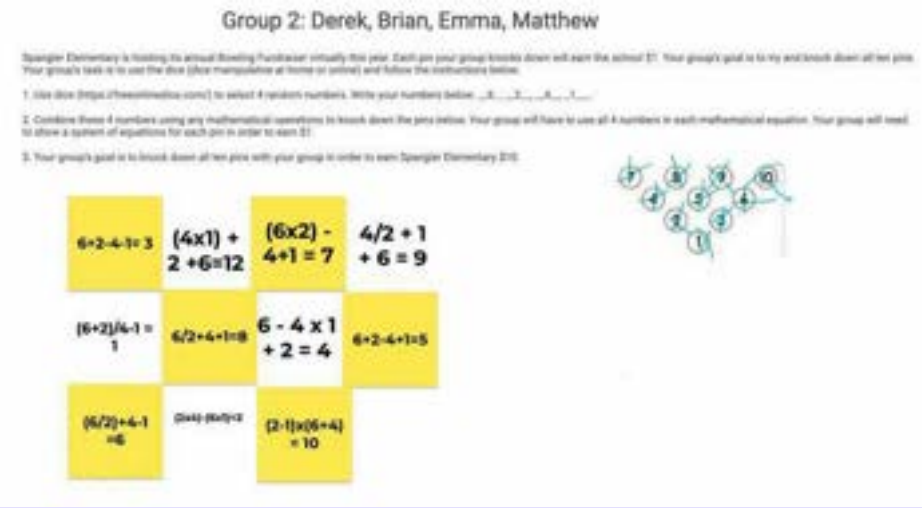
Google Jamboard is pretty necessary because when my friend or classmate doesn't understand a math problem, I use it to show different strategies on how to solve the problem. Then they understand it better.
- Alex Difuntorum (5th Grade)

Google Workspace

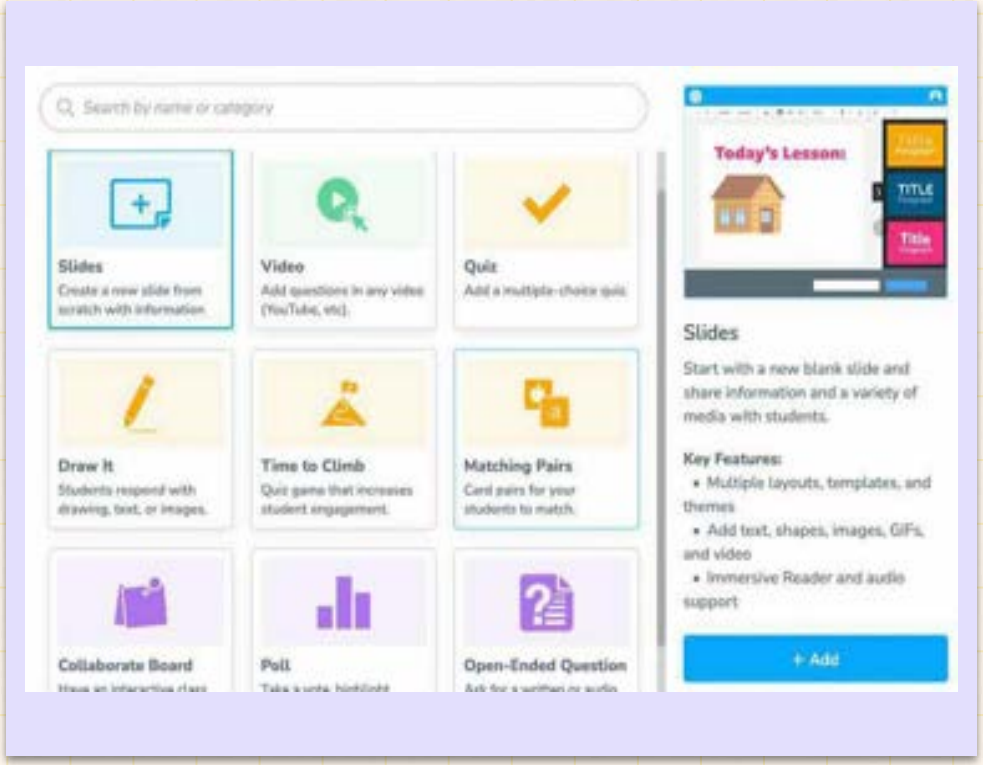
Google Classroom Technology Menu



Google Jamboard Math Task Example



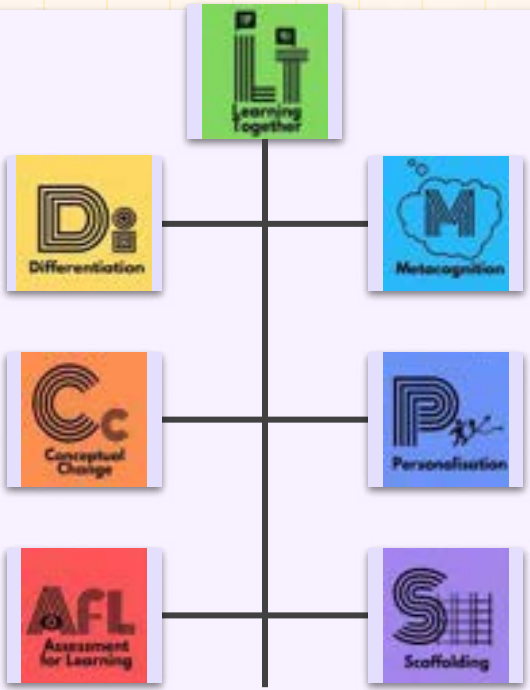
Nearpod



Nearpod Math Task Example

1. Choose from 22,000+ premade, editable lessons, videos, and activities.
2. Customize your own slide deck and add Collaborate Boards, Quizzes, Polls, Games, original PDFs/JPEGs/Videos.
3. Integrate Nearpod with Google Workspace.
4. Launch lessons for student feedback using a live lesson option (with the ability to share **live** student responses) or a student-paced option.

Why use Nearpod?



Differentiated content, learning processes, and products include watching videos, listening to audio, playing educational games, engaging with peers in virtual breakout rooms, going on virtual field trips, and speaking + writing responses.

Open-ended tasks, along with collaboration boards, allow students to reflect and communicate what they have learned, the key “aha” moments, and what they still need help on. Students are invested in their own learning outcomes.

Students have the opportunity to address alternative conceptions, ask for clarity, and build on each other’s knowledge to notice patterns and infer generalizations.

Teachers customize personalized and interactive content based on individual student needs. Students complete at their own pace.

Formative assessment activities include polls, quizzes, open-ended tasks, student-drawn responses, and collaboration boards. The formative and summative assessment allows teachers to target feedback and guide further instruction.

The simple presentation style, organization of information, and embedded digital scaffolds guide interactions and the delivery of open-ended tasks.

*from Singapore’s Ministry of Education EdTech Plan

Resources

Websites:

- https://caaspp-elpac.ets.org/caaspp/
- https://wvde.us/wp-content/uploads/2021/10/22474-Mathematical-Habits-of-Mind-PLC-Guide-for-Early-and-Elementary-v2.pdf
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- https://www.101qs.com/3675-girl-scout-cookies
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